NUMBER CONVERSION

Problem Solving with Computers-I





External vs. Internal Representation

External representation:
Convenient for programmer
Decimal (base 10)

- Internal representation:
 - Actual representation of data in the computer's memory: Always binary (1's and 0's)





Positional encoding for non-negative numbers

• Each position represents some power of the base

Binary representation (base 2)

- On a computer all data is stored in binary
- Only two symbols: 0 and 1
- Each position is called a *bit*
- Bits take up space
- 8 bits make a byte
- Example of a 4-bit number



- Actually the data is voltages
- We use the abstraction:
 - High voltage: 1 (true)
 - Low voltage: 0 (false)

$101_5 = ?$ In decimal

- A. 26
- B. 51
- C. 126
- D. 130

Converting between binary and decimal

Binary to decimal: $1 \ 0 \ 1 \ 1 \ 0_2 = ?_{10}$

Decimal to binary: 34₁₀=?₂

Hex to binary

- Each hex digit corresponds directly to four binary digits
- Programmers love hex, why?

$$25B_{16} = ?$$
 In

binary

Hexadecimal to decimal

 $25B_{16} = ? Decimal$

Hexadecimal to decimal

• Use polynomial expansion

•
$$25B_{16} = 2*256 + 5*16 + 11*1 = 512 + 80 + 11$$

= 603

• Decimal to hex: 36₁₀=?₁₆

Binary to hex: 1000111100

A. 8F0

B. 23C

C. None of the above

Numbers Binary Code

How many (minimum) bits are required to represent the numbers 0 to 3?

Colors Binary code







How many (minimum) bits are required to represent the three colors?

Characters

'a' 'b' 'c' 'd' 'e'

N bits can represent at most 2^{N} things

What is the minimum number of bits required to represent all the letters in the English alphabet in lower case?

A. 3
B. 4
C. 5
D. 6
E. 26

- Logical values?
 - $0 \Rightarrow$ False, $1 \Rightarrow$ True
- colors ?
- Characters?
 - 26 letters \Rightarrow 5 bits (2⁵ = 32)
 - upper/lower case + punctuation
 ⇒ 7 bits (in 8) ("ASCII")
 - standard code to cover all the world's languages ⇒ 8,16,32 bits ("Unicode")
 www.unicode.com
- locations / addresses? commands?
- MEMORIZE: N bits ⇔ at most 2^N things





What is the maximum positive value that can be stored in a byte?

A. 127

B. 128

C. 255

D. 256

Data types

Binary numbers in memory are stored using a finite, fixed number of bits typically:

- 8 bits (byte) 16 bits (balf word
- 16 bits (half word)
- 32 bits (word)
- 64 bits (double word or quad)

Data type of a variable determines the:

- exact representation of variable in memory
- number of bits used (fixed and finite)
 - range of values that can be correctly represented