## NUMBER CONVERSION

Problem Solving with Computers-I


## External vs. Internal Representation

- External representation:
- Convenient for programmer
- Decimal (base 10)

- Internal representation:
- Actual representation of data in the computer's memory: Always binary (1's and 0's)



## Positional encoding for non-negative numbers

- Each position represents some power of the base


## Binary representation (base 2)

- On a computer all data is stored in binary
- Only two symbols: 0 and 1
- Each position is called a bit
- Bits take up space
- 8 bits make a byte
- Example of a 4-bit number

- Actually the data is voltages
- We use the abstraction:
- High voltage: 1 (true)
- Low voltage: 0 (false)


## $101_{5}=$ ? In decimal

A. 26
B. 51
C. 126
D. 130

## Converting between binary and decimal

 Binary to decimal: $10110_{2}=?_{10}$Decimal to binary: $34_{10}=?_{2}$

## Hex to binary

- Each hex digit corresponds directly to four binary digits
- Programmers love hex, why?

$$
25 \mathrm{~B}_{16}=\text { ? } \mathrm{In}
$$

## binary

| 00 | 0 | 0000 |
| :--- | :--- | :--- |
| 01 | 1 | 0001 |
| 02 | 2 | 0010 |
| 03 | 3 | 0011 |
| 04 | 4 | 0100 |
| 05 | 5 | 0101 |
| 06 | 6 | 0110 |
| 07 | 7 | 0111 |
| 08 | 8 | 1000 |
| 09 | 9 | 1001 |
| 10 | $A$ | 1010 |
| 11 | $B$ | 1011 |
| 12 | $C$ | 1100 |
| 13 | D | 1101 |
| 14 | E | 1110 |
| 15 | F | 1111 |

Hexadecimal to decimal

## $25 B_{16}=$ ? Decimal

## Hexadecimal to decimal

- Use polynomial expansion
- $25 \mathrm{~B}_{16}=2 * 256+5^{*} 16+11^{*} 1=512+80+11$

$$
=603
$$

- Decimal to hex: $36_{10}=?_{16}$


## Binary to hex: 1000111100

A. 8 FO
B. 23 C
C. None of the above

## BIG IDEA: Bits can represent anything!!

## Numbers Binary Code

```
0
1
2
3
```

How many (minimum) bits are required to represent the numbers 0 to 3 ?

## BIG IDEA: Bits can represent anything!!

## Colors <br> Binary code

Red

## Green

Blue

BIG IDEA: Bits can represent anything!!
Characters
'a'
'b'
'c'
'd'
'e’
$N$ bits can represent at most $2^{N}$ things

# What is the minimum number of bits required to represent all the letters in the English alphabet in lower case? 

A. 3
B. 4
C. 5
D. 6
E. 26

## BIG IDEA: Bits can represent anything!!

- Logical values?
- $0 \Rightarrow$ False, $1 \Rightarrow$ True


## Red

Green

- colors ?
- Characters?
- 26 letters $\Rightarrow 5$ bits ( $2^{5}=32$ )
- upper/lower case + punctuation
$\Rightarrow 7$ bits (in 8) ("ASCII")
- standard code to cover all the world's languages $\Rightarrow 8,16,32$ bits ("Unicode") www.unicode.com
- locations / addresses? commands?
- MEMORIZE: N bits $\Leftrightarrow$ at most $2^{\mathrm{N}}$ things

What is the maximum positive value that can be stored in a byte?
A. 127
B. 128
C. 255
D. 256

## Data types

Binary numbers in memory are stored using a finite, fixed number of bits typically:

8 bits (byte)
16 bits (half word)
32 bits (word)
64 bits (double word or quad)

Data type of a variable determines the:

- exact representation of variable in memory
- number of bits used (fixed and finite)
- range of values that can be correctly represented

